Robust Control of Flexible Spacecraft

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A well-designed feedback control system exhibits the properties of external disturbance attenuation and performance robustness with respect to plant uncertainty. The plant uncertainties of flexible spacecraft include unmodeled dynamics and parameter uncertainties. Singular value robustness measures are used to compare performance and stability robustness properties of different control design techniques in the presence of residual modal interaction (control and observation spillover) for a design example which is representative of a practical flexible spacecraft system. The control designs evaluated include linear quadratic geometry (LQG) control, integral feedback, bias removal control, innovations feedthrough, and frequency-shaped LQG.

Introduction

THE control system for a flexible spacecraft must exhibit performance robustness to a variety of disturbances and uncertainties. The key problem areas are:

1) Controlling an infinite dimensional system with a finite dimensional controller.

2) The effect of actuator and sensor location, number, and dynamics.

3) Uncertain system parameters and unmodeled modes, particularly those within or near the controller bandwidth.

4) Internal and external disturbances.

It is fundamental to flexible spacecraft control that all of these problem areas be addressed in the control design method and in the evaluation of closed-loop performance. Any of these factors alone or in combination can significantly degrade performance and even cause instability. Typically, tradeoffs among bandwidth, accuracy, and actuator/sensor characteristics and locations must be made to accommodate the above requirements for a particular spacecraft structure and application.

The main objective of present control design techniques for flexible structures is to guarantee performance and stability robustness in the presence of uncertain unmodeled dynamics, e.g., the "spillover" effect. This paper shows how different control design methods may be evaluated using input/output robustness tests, irrespective of assumptions inherent in each design methodology. The robustness tests are based on the conic sector conditions of the small gain theorem.[1-4]

The fundamental control design problem for flexible spacecraft is model uncertainty—not only uncertain parameters which are modeled, but uncertain unmodeled dynamics. This situation arises because flexible spacecraft structures are coupled systems of elastically deformable bodies whose behavior is characterized by nonhomogeneous equations with uncertain parameters. Approximate solutions are required to solve them. The approximate numerical models are not sufficiently accurate and data obtained from on-Earth testing is often incomplete.

A typical frequency response of an identified structure, including its error bounds, is shown in Fig. 1.§ Al low frequencies, the model parameters are uncertain; at high frequencies, the model exhibits large errors which are due to unmodeled dynamics. These types of model uncertainties are referred to as structured and unstructured uncertainties, respectively.[4]

Reference 5 is a survey of the large space structures control problem that summarizes most of the current design methods proposed over the past five years. Current methods have primarily addressed the problem of "spillover" reduction, i.e., reducing the deleterious effects of unmodeled modes and proper selection of retained modes. The effect of disturbances and the effect of actuator and sensor dynamics have not received the same attention, nor has the problem of model uncertainty. The problem of spillover, however, is still not totally resolved. Consequently, for the spacecraft control problem where there are many highly uncertain flexible modes and high sensitivity to the interactions among actuator and sensor dynamics and disturbances with critical modes, it is necessary to be able to quantify the "robustness" of the control design. The methodology presented in this paper addresses the problem of quantifying robust control for flexible spacecraft. Frequency domain robustness tests are developed that provide quantitative measures of robustness which can be used to evaluate and modify designs. In particular, LQG modal control[5,6] is examined, including typical variations such as integral control, bias removal, and frequency shaping[7] of the cost functional.

Reduced-Order Control Design

The starting point for control design is a high-order model which represents the flexible spacecraft. A popular choice is a finite-element model (FEM) (obtained from NASTRAN-type codes), which typically contain several thousand coupled ordinary differential equations of various degrees of significance and accuracy. While modern control design methods apply in theory to systems of any order, control design algorithms are limited by computer wordlengths, speed, and accuracy of iterative procedures, thereby setting dimensional limits. Control implementation is further constrained by the number and type of actuators and sensors that are economically feasible.

The fundamental issue is that the system order is too large and must be reduced. In a typical design procedure the FEM is reduced to a lower order model, which we will refer to as the finite-design model (FDM). The reduction technique from the FEM to the FDM varies but usually is accomplished by a frequency truncation, singular value decomposition, or a modal cost analysis (MCA).[8] The order of the FDM is determined by the maximum order of a solvable Riccati or Lyapunov equation (around 150 states or 75 modes).

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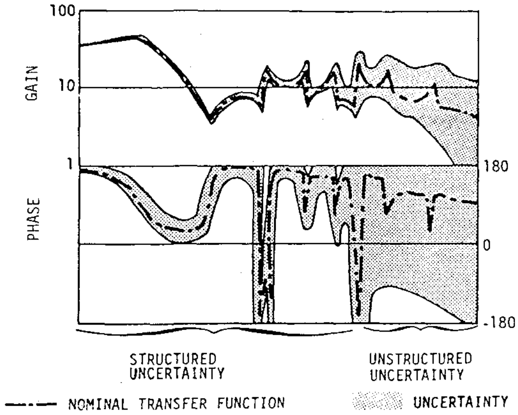

Fig. 1 Gain/phase vs frequency for flexible spacecraft.

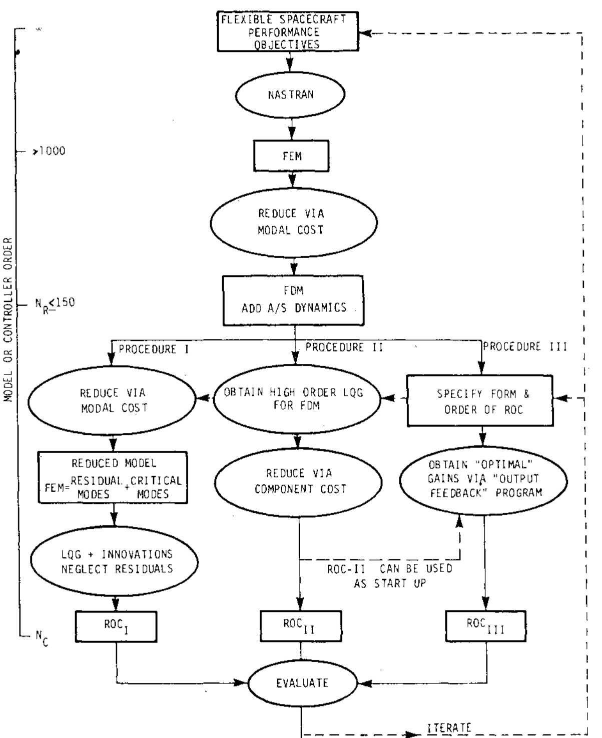

Fig. 2 Reduced-order control design approaches.

The goal of any design effort must be the development of a finite-dimensional controller of even much lower order than the FDM. This controller is referred to as a reduced-order controller (ROC). There is no unique ROC; obviously, many factors influence the form and order of the ROC. Data processing capabilities of onboard computers limit the order. Large spacial distances will limit the use of centralized information, thus requiring a decentralized control. This will limit the form as well as the order of the control. Decentralized control is an important issue in control theory and there is much literature on the subject. However, there are no current results developed specifically for large space structures (LSS).

Figure 2 shows three generic design procedures, which result in a reduced-order controller, denoted ROC-I, ROC-II, and ROC-III. Common to all design procedures is the modeling of the elastic spacecraft by a high-order FEM, the introduction of sensor and actuator dynamics and disturbance models, the subsequent reduction to a lower order FDM, (possibly) a cost functional to be optimized, and an evaluation process. The cost functional may also affect the model reduction procedure as a means to determine the appropriate reduction, e.g., MCA. Typical ROC design approaches which fit the generic categories in Fig. 2 are described below.

ROC-I. Open-Loop Order Reduction

Step 1: Reduce the FDM into primary and residual modes, based on MCA and/or engineering judgement (e.g., modes that are unstably interacting[12] are kept).

Step 2: Neglect the residual modes, add sensor and actuator dynamics, and design an LQG control for the primary modes.

In this approach, a simplified version of the LSS is obtained by decomposing the FDM into primary and residual modes x_p and x_r, respectively, where

$$\dot{x}_p = A_p x_p + B_p u \qquad \dot{x}_r = A_r x_r + B_r u \tag{1}$$

The primary modes x_p are those critical for control. Neglecting the residual model for design purposes, the control may then be selected to have the "observer"-based form

$$u = -F\hat{x}_p \qquad \dot{\hat{x}}_p = A_p \hat{x}_p + B_p u + K(y - C_p \hat{x}_p) \tag{2}$$

The control gains (F,K) can be selected using either LQG theory or pole placement. Obviously, this control cannot guarantee stability in the FEM due to spillover. To reduce the spillover in the residual modes, a direct output feedback term can be added so that,

$$u = -F\hat{x}_p - Dy \tag{3}$$

where D is designed as a collocated rate feedback, which by itself guarantees stability of the FEM. The LQG modal control [Eq. (2)] is sometimes referred to as a high-authority control (HAC), whereas the Dy term in Eq. (3) is a low-authority control (LAC).[9] As pointed out by Canavin,[10] the purpose of the LAC is to reduce the effect of spillover in the residual modes.

A similar approach was suggested by Balas.[6] In this approach the modal representation is decomposed into primary, secondary, and residual modes, x_p, x_q, and x_r, respectively, where

$$\dot{x}_p = A_p x_p + B_p u \quad \dot{x}_q = A_q x_q + B_q u \quad \dot{x}_r = A_r x_r + B_r u \tag{4}$$

The modal control has an added innovations feedthrough term where the innovations are generated from a modified observer

$$u = -F\hat{x}_p - D(y - \hat{y}), \qquad \hat{y} = C\hat{x}_p$$

$$\dot{\hat{x}}_p = A_p \hat{x}_p + (B_p + TB_q)u + K(y - \hat{y}) \tag{5a}$$

where T and D are solutions of

$$(A_p - KC_p)T - TA_q + KC_q = 0 \qquad D(C_pT - C_q) - FT = 0 \tag{5b}$$

If $(A_p - KC_p)$ and A_q have no common eigenvalues and if $\dim(y) = \text{rank}\ (C_pT - C_q) = \dim\ (x_q)$, then (T,D) can be uniquely determined from Eq. (5b) so that the closed-loop eigenvalues of Eqs. (4) and (5a), neglecting the residual modes x_r, are those of $(A_p - B_pF)$, $(A_p - KC_p)$, and A_q.

This control does not guarantee closed-loop stability. What it does is remove spillover from the q-secondary modes at the expense of possibly increasing spillover in the r-residual modes. However, the r-residual modes may attenuate the spillover more so than in the q modes because of higher natural damping.[19] The selection of the q modes is dependent on the designer's judgment as to where spillover is a problem. The number of q modes is dependent on the number of sensors.

ROC-II. Closed-Loop Order Reduction

Step 1: Design full order LQG control for FDM with sensors and actuators.

Step 2: Reduce design of Step 1 via MCA, engineering judgment, etc.

In this approach the full-order LQG control [Eq. (2)], together with the FDM, is reduced to lower order simultaneously. The result is a reduced-order control which guarantees stability of the FDM.

ROC-III. Direct ROC Design

Step 1: Specify form and order of ROC.

Step 2: Determine ROC gains directly.

In this approach the order and form of the ROC are all that need be specified. For example, a typical controller form is,

$$u = -F_{11}x_c - F_{12}y \qquad \dot{x}_c = F_{21}x_c + F_{22}y \tag{6}$$

where preselected free elements in F_{ij} (e.g., gains, time constants, etc.) are determined by output feedback pole placement or optimal output feedback.[11,12] Passivity-based approaches[13] can also be included in this category provided that the controller utilizes colocated rate sensors and force actuators.

Discussion

There are advantages and disadvantages to each of the three types of generic approaches. ROC-I is the easiest to compute because the control gains are determined from a relatively low-order model. However, model reduction is based on open-loop performance; consequently, closed-loop stability is not guaranteed in the presence of the residual modal interaction.

ROC-II does guarantee stability because the model reduction is based on closed-loop performance. However, this method requires starting with a high-order (100 modes) LQG solution. In addition, the resulting reduced-order control, although stable, is suboptimal for that order of control. Furthermore, if an LQG design is used, the controller may still be too complex for practical implementation.

ROC-III guarantees both stability and optimality for a specified form and order of control. However, the optimal gains must be computed so as to solve a highly nonlinear system of algebraic equations. Although algorithms converge, there is no guarantee that the solution is the global optimum.[11]

Common to all of the ROC methods is a high-order finite-dimensional model of the flexible spacecraft with many uncertain parameters. None of the methods account specifically for the simultaneous effect of reduced-order modeling (spillover) *and* parameter uncertainty.

Most control design methods propose the use of a filter or observer to estimate the modal states. Design of the filter (and controller gains) using LQG theory results in a control that is optimal with respect to a quadratic performance measure and the assumed *model* of the LSS. However, measures of optimality are not necessarily measures of robustness (see, for example, Refs. 3, 4, 14, and 15). Consequently, although a variety of candidate control structures may yield similar quadratic measures of optimality, their robustness properties may be quite dissimilar.

Robustness of Reduced-Order Control

A general theory of the robustness of multivariable feedback systems[1-4] has shown that perturbed unity feedback systems with additive or multiplicative perturbations (as shown in Figs. 3a and 3b, respectively) remain stable if, for additive perturbations

$$\bar{\sigma}[L(j\omega)] < \underline{\sigma}[I + G_0(j\omega)], \quad \omega > 0 \tag{7}$$

or for multiplicative perturbations

$$\bar{\sigma}[L(j\omega)] < \underline{\sigma}[I + G_0^{-1}(j\omega)], \quad \omega > 0 \tag{8}$$

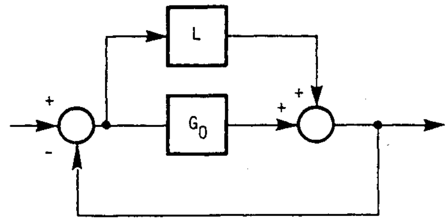

Fig. 3a Additive uncertainty.

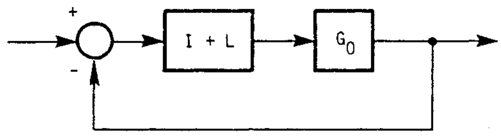

Fig. 3b Multiplicative uncertainty.

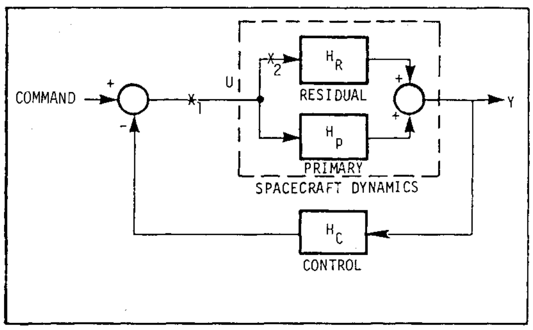

Fig. 4 Controlled spacecraft.

provided that the nominal feedback system ($L \equiv 0$, no uncertainty) is stable, and the perturbation L is also stable. ($\bar{\sigma}(A)$ and $\underline{\sigma}(A)$ represent the maximum and minimum singular values, respectively, of the complex matrix A; the singular values are the square roots of the eigenvalues of A^*A.) These tests can be applied to establish robustness properties of control systems designed for flexible spacecraft control.

Consider the controlled spacecraft system shown in Fig. 4. The controller is represented by the transfer function matrix H_c. (The dependent variable s or $j\omega$ is suppressed for brevity of notation and is shown explicitly, if necessary.) The spacecraft is decomposed into two subsystems: H_p, the primary (modeled modes) system, and H_r, the residual (unmodeled modes) system. In order to apply the robustness tests of Eqs. (7) and (8), we need to determine what part of the system (Fig. 4) corresponds to the uncertain dynamics L and what part corresponds to the nominal loop gain G_0. This depends on the test data available about the system. The following test is based on the uncertainty in the residual dynamics H_r, which can be isolated by breaking the loop¶ at X_2 to give *robustness test 1*

$$\bar{\sigma}(H_r) < 1/\bar{\sigma}[H_c(I + H_pH_c)^{-1}] = \underline{\sigma}(H_c^{-1} + H_p) \tag{9}$$

provided H_c^{-1} exists. [In Eq. (9), we have used the fact that $\bar{\sigma}(A^{-1}) = 1/\underline{\sigma}(A)$.]

If the number of sensors and actuators are equal and the plant is minimum phase and invertible (H_p^{-1} exists), then G may be written as,

$$G = H_cH_p(I + H_p^{-1}H_r) \tag{10}$$

¶"Breaking the loop" is the usual engineering jargon for determining a return difference transfer function matrix[4] at a specified node in the loop, e.g., $(I + G_0)$ in Eq. (7) or $(I + G_0^{-1})$ in Eq. (8).

which, when compared with Eq. (8), identifies $H_p^{-1}H_r$ as L and H_cH_p as G_0. This gives *robustness test 2,*

$$\bar{\sigma}(H_p^{-1}H_R) < \underline{\sigma}[I + (H_cH_p)^{-1}] \tag{11}$$

provided $H_p^{-1}H_r$ is stable.

The tests require determining $\bar{\sigma}(H_r)$ or $\bar{\sigma}(H_p^{-1}H_r)$, respectively, as a function of frequency. Experimental procedures can be devised in each case. For example, an upper bound on $\bar{\sigma}(H_p^{-1}H_r)$ can be obtained experimentally as follows: inject sinusoidal signals $u(t)$ of frequency ω into the spacecraft actuators; calculate the error $e(t) = y(t) - y_p(t)$, where $y(t)$ is the output of the spacecraft sensors and $y_p(t)$ is the output of the primary (modeled) modes, i.e., $y_p = H_p u$ as in Fig. 4. It can be shown that $\|e\|/\|y_p\|$ is an upper bound on $\bar{\sigma}(H_p^{-1}H_r)$ at each frequency ω, where $\|\cdot\|$ is the rms norm defined as

$$\|x\| = \left(\frac{\omega}{2\pi}\int_0^{2\pi/\omega} x'(t)x(t)\mathrm{d}t\right)^{1/2}$$

A similar procedure is necessary to obtain $\bar{\sigma}(H_r)$ in Eq. (9), i.e., $\|y - y_p\|/\|u\|$ bounds $\bar{\sigma}(H_r)$. These bounds are used in the examples to follow.

Robustness tests 1 and 2 are determined by loop breaking at the input to the system. Breaking the loop at the output, e.g., a sensor uncertainty, changes the test. For example, in the right hand side of Eq. (11), the term H_cH_p would be replaced with H_pH_c. In a scalar system, there is no difference, but in a multivariable system the difference can be considerable. For flexible structures with colocated actuators and sensors, there is an inherent modal symmetry which causes no difference in the input or output (actuator or sensor) uncertainty robustness tests. Reference 16 using a different loop breaking point gives a more detailed description for a shape/vibration LQG modal control using only position measurements.

Robustness of LQG Modal Control

The spacecraft state model [Eq. (1)] together with LQG modal control [Eq. (2)], as shown in Fig. 4, has the transfer functions,

$$H_p(s) = C_p(sI_p - A_p)^{-1}B_p,\quad H_r(s) = C_r(sI_r - A_r)^{-1}B_r$$

$$H_c(s) = F(sI - A_c)^{-1}K, \qquad A_c = A_p - B_pF - KC_p \tag{12}$$

This control guarantees the stability of the nominal system H_cH_p as long as the eigenvalues of $(A_p - B_pF)$ and $(A_p - KC_p)$ have negative real parts. The effect of uncertainty in H_r can be evaluated by using the above tests. Note that uncertainty in the residual system H_r can be either parameter uncertainty or structural failures. Consequently, the robustness tests can give an indication of structural reliability, which is an important aspect of control design.

There may be difficulties in applying robustness tests 1 and 2 with LQG modal control. These tests require H_c to be stable. With LQG control *there is no guarantee that H_c is stable.* This is a source of robustness problems with LQG control. Reference 17 gives an in-depth description of this problem. If H_c is unstable and a control loop or a sensor becomes disabled, the system will most likely be unstable. Also, changes in the true system model (A,B,C) matrices may cause instability. This results because no measure of robustness has been intentionally designed into the system.

However, it has been shown[3] that the full-state feedback linear quadratic (LQ) control,

$$u = -Fx \tag{13}$$

with F from Eq. (2), guarantees stability for one-half to infinite gain margin per channel, or 60 deg phase margin per channel. Unfortunately, this guarantee is too restrictive in regard to robustness and other practical considerations of multiloop control. First of all, LQ control requires full-state feedback into all input channels. Generally this is impossible, since all of the states are not available for measurement. Second, and most important to robustness, parameter variations are not likely to appear only in the form of gain or phase perturbations.

An even more distressing situation arises when utilizing observer-based (LQG) control. Although performance is only slightly degraded, there are no guaranteed margins at all![4,14] However, the observer gain K can be adjusted,[14] under certain conditions, so that the gain and phase margins of the LQG control approach asymptotically the gain and phase margin properties of the LQ control. But this does not mitigate the robustness problem since gain/phase margins, as already mentioned, are not necessarily measures of robustness. In fact, the inadequacy of gain/phase margin properties to reflect robustness properties is true, in general, for mutivariable controllers and not limited to LQ or LQG controllers.[15]

Robustness to Actuator/Sensor Dynamics

The frequency domain robustness tests [Eqs. (7) and (8)] can be used to examine the effect of uncertainty in actuator/sensor dynamics. In the system of Fig. 4, add actuator dynamics H_a and sensor dynamics H_s to the input and output of the spacecraft dynamics, respectively. Suppose that H_a is perturbed by an additive perturbation δH_a. Breaking the loop at the input to the actuator perturbation δH_a gives for *robustness test 3,*

$$\bar{\sigma}(\delta H_a) < 1/\bar{\sigma}[H_cH_sH(I + H_aH_cH_sH)^{-1}] \tag{14}$$

with

$$H = H_p + H_r \tag{15}$$

Other tests result if the actuator perturbation is multiplicative, i.e., $(I + \delta H_a)H_a$, rather than additive as in the above test. A similar battery of tests can also be obtained for sensor perturbations.

These results have some interesting consequences. Suppose that for the nominal system, no other actuator/sensor or compensator gives better performance. In this case, the robustness test can evaluate candidate actuators which are not optimal. Tradeoffs in actuator parameters and robustness are easily established without the need for recalculating a new optimal control.

The effect of actuator/sensor parameter uncertainty and actuator/sensor failure can also be tested. If, uncertainty results from actuator/sensor failure, the tests give an indication of reliability to failures in control/communication channels that are not catastrophic, i.e., do not destabilize.

Examples

This section illustrates the use of robustness tests 1-3 by application to a representative controlled flexible spacecraft. Six LQG-based generic control designs are compared: modal, integral, bias removal, acceleration, innovations feedthrough, and frequency shaped. Due to space limitations, the forms of the LQG-based control designs are not shown here; any good textbook can be consulted (for example, Ref. 24). (Complete examples appear in Refs. 19 and 20.) These designs all represent elements in typical design procedures and as such cannot be used to compare different design philosophies. For example, it would be of considerable interest to compare the robustness properties of such known approaches as the LMSC HAC/LAC approach,[9] the General Dynamics MESS algorithm,[21] and Meirovitch's IMSC approach,[22] to name a few.

The flexible spacecraft model developed by the Charles Stark Draper Laboratory[18] is a tetrahedral truss structure which is supported at the base by three right-angle bipods. The individual truss members, including the bipods, are

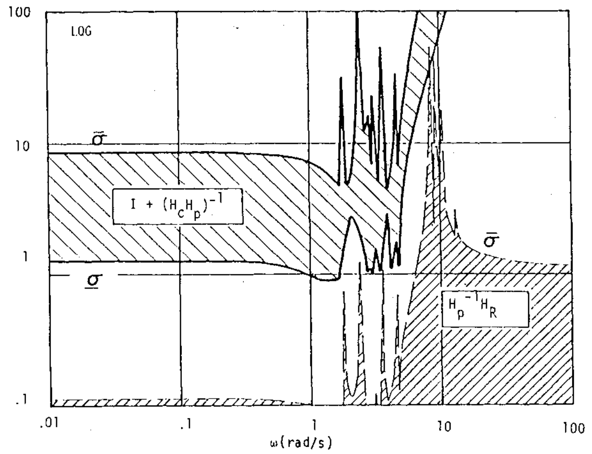

Fig. 5a Robustness test 2 for LQG control.

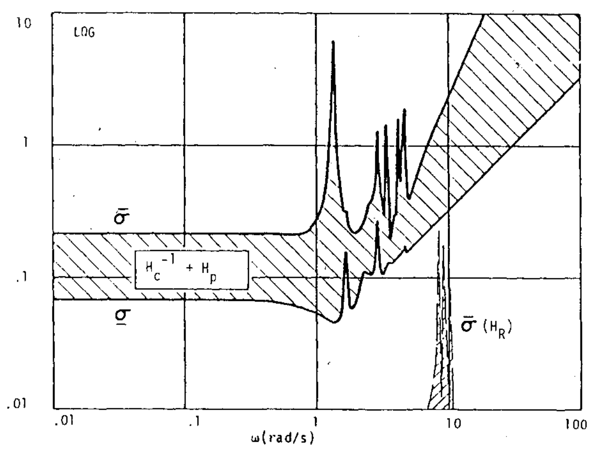

Fig. 5b Robustness test 1 for LQG control.

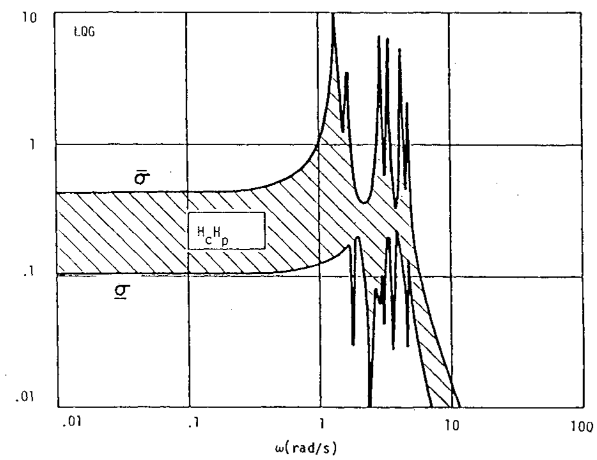

Fig. 5c Loop gain for LQG control.

flexible in their axial direction only. The model is representative of many radar and optical control problems. The nominal model, used for simulation and evaluation, has 12 modes. For the control design, only eight modes are assumed to be known. The four remaining modes are to be considered unknown. The objective is to control the line of sight (LOS) of the top vertex with sensors and actuators in the bipods.

LQG Modal Control

A baseline LQG modal control design was selected which minimizes a quadratic cost with weighting on controls and line-of-sight contributions of primary modes. Thus, the control has the form as described by Eq. (2). Figure 5a shows

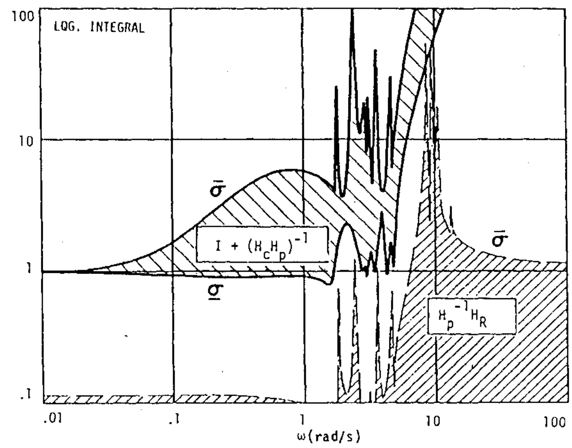

Fig. 6a Robustness test 2 for integral control.

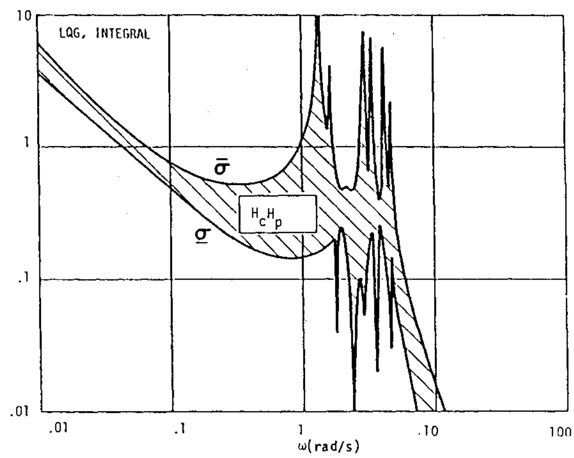

Fig. 6b Loop gain for integral control.

that robustness test 2 is not satisfied for this design technique. However, Fig. 5b shows that robustness test 1 is satisfied; thus the spacecraft is stable. Figure 5c shows the upper and lower singular values of the loop gain H_cH_p. The lower singular value of the loop gain is small for this control design (0.1), demonstrating poor tracking and disturbance rejection properties.

Many interesting phenomena are observed from these singular value plots. For example, the maxima of the upper singular values of the robustness tests (Figs. 5a and 5b) and the minima of the lower singular value of the loop gain occur at the locations of the transmission zeros of the plant.

Integral Control

To improve the steady-state performance of the LQG control design, multivariable integral control may be added to the design. Figures 6a and 6b depict robustness test 2 and loop gains, respectively, for output integral control. The upper singular value of the robustness measure in Fig. 6a is close to unity and the lower singular value of the loop gain in Fig. 6b is large at low frequencies, indicating good steady-state characteristics. The high-frequency characteristics are similar to that of the LQG control design, however, robustness test 2 is not satisfied at high frequencies.

Bias Removal Control

To remove a constant input bias b the equation, $\dot{b}=0$, may be added to the observer. This approach is useful for handling certain types of modeling errors. Figure 7 illustrates robustness test 2 which indicates that performance is well within requirements at low frequencies, i.e., high gain. However, the test for stability is not satisfied at high frequencies.

Acceleration Feedback

With acceleration feedback, Fig. 8 depicts robustness test 2. The low-frequency characteristics are similar to the LQG control design (Fig. 6a). Due to acceleration feedback, the gain increases with the square of frequency. The high gain at high frequencies decreases the robustness of the control design against spillover. Hence, against popular belief, this control design should not be chosen for flexible structure control.

Innovations Feedthrough

Following Ref. 6 and Eqs. (4) and (5), the ninth mode was selected as the secondary mode [x_q in Eq. (4)]. The control has the form given by Eq. (5). Figure 9 depicts robustness test 2 for this control design. The output spillover of the secondary mode is removed (no stability problems at 8.54 rad/s). The robustness of the system is reduced significantly (compare with LQG design in Fig. 6a). This demonstrates the danger of separate treatment of individual states in multidimensional problems. It can be shown that this control design places blocking zeros at the frequency of the critical residual mode between the output of the secondary mode and the output of the compensator. These blocking zeros are not structurally robust zeros.[23]

Frequency-Shaped Control

Reference 7 proposes to use a frequency-weighted cost functional to reduce the gain of the compensator at high frequencies. The quadratic cost in the frequency domain is given by

$$J=\frac{1}{2\pi}\int_{-\infty}^{\infty}[q\|y_{\mathrm{LOS}}(j\omega)\|^2+(r_0+\omega^2 r_l)\|u(j\omega)\|^2]\,\mathrm{d}\omega \qquad (16)$$

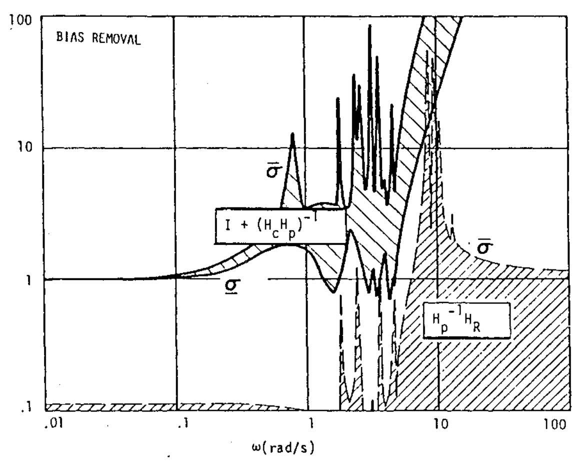

Fig. 7 Robustness test 2 for bias removal control.

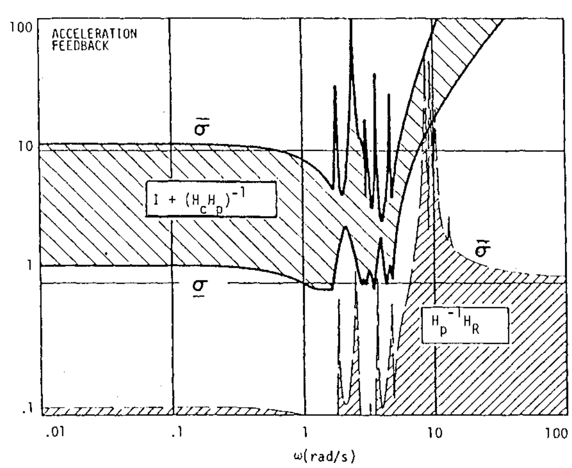

Fig. 8 Robustness test 2 for acceleration control.

The control energy is penalized more heavily at higher frequencies where robustness is a problem due to the unmodeled residual modal interaction. Figure 10 depicts the stability measures for this control design. The lower singular value of the robustness measure is greater than the upper singular value of the dynamic uncertainty at all frequencies. The control design is therefore stable. However, at low frequencies, the loop gain is too small (≈ 0.01) to satisfy any reasonable performance requirements.

Review of Results

The robustness tests give an indication of the advantages and disadvantages of each control design with respect to

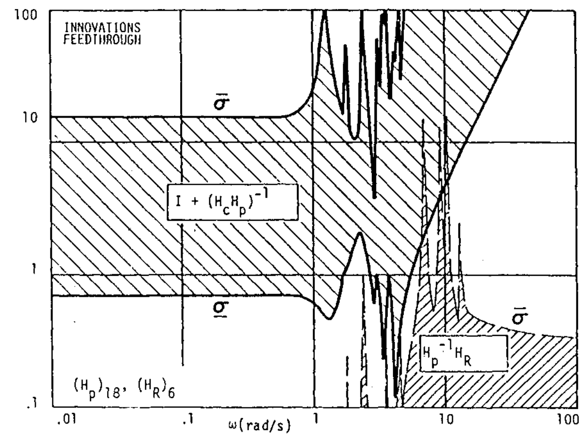

Fig. 9 Robustness test 2 for innovations feedthrough control.

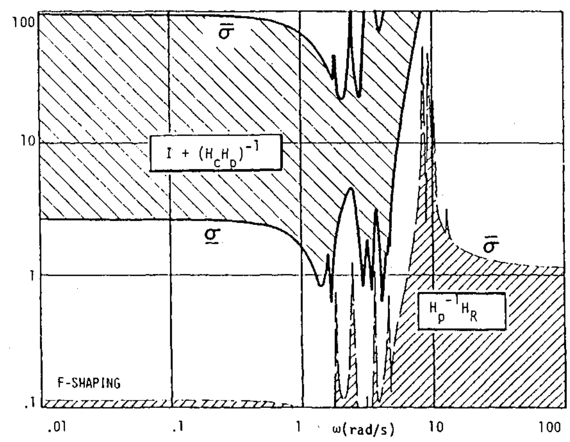

Fig. 10 Robustness test 2 for frequency-shaped control.

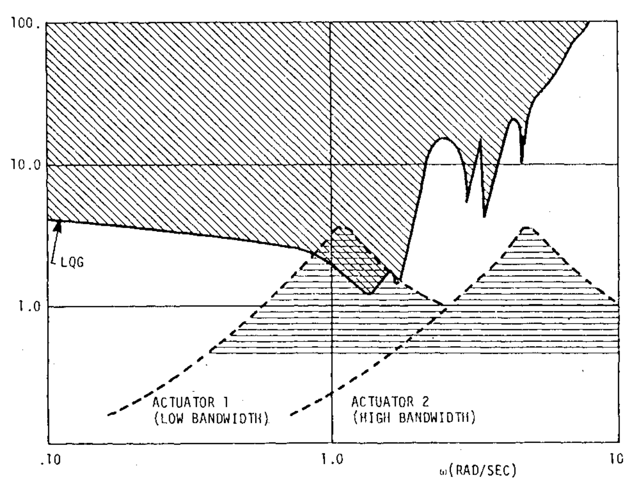

Fig. 11 Robustness test 3 for actuator bandwidth perturbations.

performance and stability robustness. Integral and bias removal control satisfy the loop gain requirements, but are not robust. Frequency-shaping control is robust at high frequencies but does not satisfy low-frequency performance requirements. LQG control has low gains at low frequencies and is not robust at high frequencies.

None of the control design methods provides both high gains in the desired frequency range and is stability robust. A frequency-shaped weighting should result in a performance and stability robust control. For example, if q in Eq. (16) is replaced with $(q_0 + q_1/\omega^2)$, then the closed-loop system will exhibit integral action at low frequencies, as well as stability robustness at high frequencies due to the control penalty $(r_0 + r_1\omega^2)$.

Actuator Uncertainty

Typical band-limited actuators may be described by the second-order system,

$$H_a = \frac{\omega_a^2}{s^2 + 2\zeta_a\omega_a s + \omega_a^2} \tag{17}$$

For identical actuators in each channel, the actuator uncertainty [Eq. (14)] is

$$\delta H_a = I - H_a = \frac{s(s + 2\zeta_a\omega_a)}{s^2 + 2\zeta_a\omega_a s + \omega_a^2} I \tag{18}$$

with upper singular value

$$\bar{\sigma}[\delta H_a(j\omega)] = \frac{\omega(\omega^2 + 4\zeta_a^2\omega_a^2)^{1/2}}{[(\omega^2 - \omega_a^2)^2 + 4\zeta_a^2\omega_a^2\omega^2]^{1/2}} \tag{19}$$

Two items determine the actuator bandwidth requirements: 1) a low actuator bandwidth can reduce the robustness of the system and can even cause instability (actuator 1 of Fig. 11); and 2) a high actuator bandwidth can provide frequency shaping (i.e., reduce the control forces at high frequencies) and increase the robustness of the system against residual mode spillover (actuator 2 of Fig. 11).

For this example, the actuator requirement can easily be established by shifting the actuator perturbation along the ω axis of the return difference sigma plot of the LQG control design (see Fig. 11). It is clear that the required bandwidth of the actuator is a function of the compensator bandwidth.

Conclusions

Frequency domain stability robustness tests have been used to evaluate the robustness of control systems for flexible spacecraft. This is a particularly useful tool in multivariable control design and parallels Bode plots for single-input/single-output systems.

A flexible space structure control example shows the significance of these robustness tests for the analysis of stability and performance with respect to spillover, as well as measures for actuator selection. Several generic control design methods were analyzed for the selected design problem. The graphical representations show the robustness properties of each control design, demonstrating the value of these tools to the control designer.

The robustness properties as exhibited are somewhat dependent on the robustness test used, e.g., Fig. 5a (robustness test 2) indicates poor robustness whereas Fig. 5b (robustness test 1) does not. In general, selecting the appropriate test requires engineering judgement. A good rule of thumb is to isolate the uncertain part (as in Fig. 5b), but this requires specific test data which may be costly to obtain. On the other hand, a generalized test, e.g., robustness test 2, at least highlights potential worst case situations. This is extremely valuable information, particularly in analyzing a complex spacecraft system.

At present, additional research is required to draw strong conclusions. However, some form of integral control or bias removal control appears necessary to give low-frequency performance. High-frequency robustness requirements need some form of frequency shaping. Methods to improve the shaping function are still needed. Also, design techniques which provide robustness with respect to specific parameter uncertainties are lacking.

Further, the main issue addressed in this paper is stability robustness from which performance (e.g., tracking/pointing) robustness is implied. Direct approaches to performance robustness evaluation (e.g., Ref. 15), need to be investigated for flexible spacecraft.

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